Estimating river discharge from the Surface Water and Ocean Topography mission:
Estimated accuracy of approaches based on Manning's equation

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Introduction

Many rivers worldwide are **currently unmonitored**. Widespread installation and maintenance of traditional river gages is either economically or physically infeasible. As such, monitoring of the world's freshwater by satellite remote sensing is an attractive supplement to the in situ river gage network. The Surface Water and Ocean Topography (SWOT) mission will measure water surface elevations (WSE), water surface slope, and the areal extent of lakes, wetlands, reservoirs, floodplains, and rivers globally.

Ultimately, SWOT should provide enough information from which to estimate **instantaneous river discharge for moderately large rivers (at least 100 m wide)**. Although multiple algorithms of varying complexity are being developed for river discharge estimation, a simple approach is the application of **Manning's equation. The error implications of applying Manning's Equation to SWOT measurements are considered here.**

SWOT Measurement Accuracy Requirements

SWOT will measure water surface slope, water surface elevation (WSE) and areal extent of surface water globally (between 78°N and 78°S latitude). Measurements at most locations will be made at least twice in a 22-day repeat period. For rivers the science requirements are as follows:

Measurement	Required Accuracy (1σ)*
Slope	1 cm/km, over 10 km downstream distance inside river mask
WSE	10 cm, averaged over 1 km² area within river mask
Area	20% for all rivers at least 100 m wide

* See SWOT Science Requirements Document, available at http://swot.jpl.nasa.gov/mission/

Errors in Derived Quantities

Width

Width estimates from the SWOT water mask will be limited by classification errors, estimation of which is still in development. Early investigations (Moller et al., 2008) show that the effect of 20 ms water coherence time on relative width errors can be reduced from ~7% averaged over a 100 m long reach to ~4% averaged over reaches between 1-2 km in length. They also found that as decorrelation approaches infinity, finite pixel sizes provide a lower bound on width bias (~10 m).

Water Depth^T

SWOT will measure WSE relative to a given datum. Manning's equation requires water depth, which is equal to the difference between WSE and channel bathymetry. Since channel bathymetry at most locations is unknown, water depth will have to be derived from available observations. One possible approach, outlined by Durand et al. (2009), applies the continuity and kinematic assumptions to estimate initial water depth from width, temporal change in water depth (change in WSE) about this initial value, and slope from SWOT. Assuming that Manning's n is known from ancillary data, Durand et al. (2009) estimated depth for a model of the Ohio River with a mean relative error of 4.1% and standard deviation of relative error of 11.2 %.

Manning's Roughness (n)

Manning's roughness is generally calibrated from field measurements or estimated visually in situ; however, some efforts have been made to estimate n from channel form. The following regressions are rewritten here in terms of SWOT observables assuming a rectangular cross-section:

Riggs (1976): $n=0.210w^{-0.33}z^{0.33}s^{0.095}$

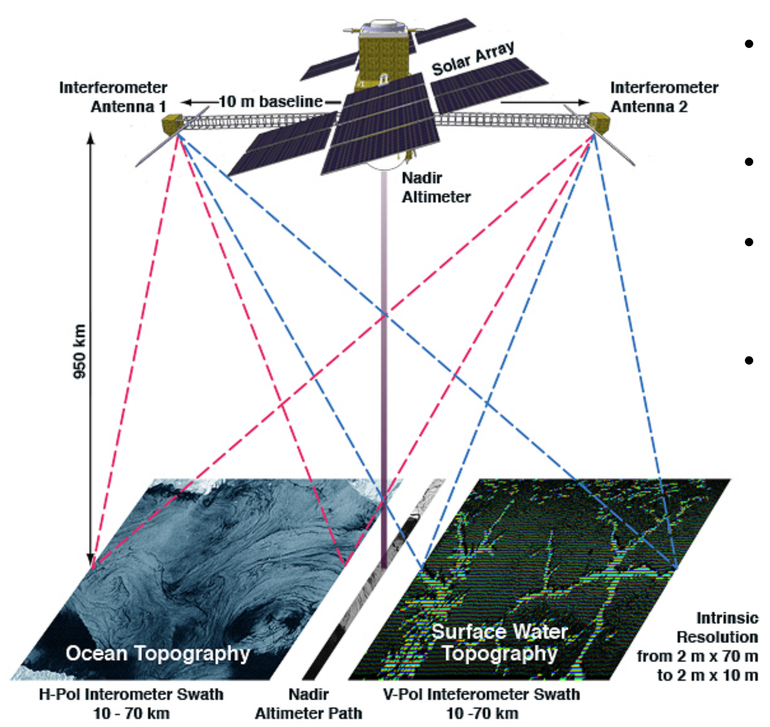
Jarrett (1984): $n=0.32z^{-0.16}s^{0.38}$

Dingman and Sharma (1997): $n=0.217w^{-0.173}z^{0.094}s^{0.156}$

Bjerklie et al. (2005) Model 1: $n=0.139w^{-0.02}z^{-0.073}s^{0.15}$

Errors associated with these regressions are estimated in Figure 1.

SWOT Mission Overview



- Ka-band SAR interferometric system with 2 swaths, 60 km each
- WSOA and SRTM heritage
- Produces heights and co-registered all-weather imagery
- Additional instruments:
  - conventional Jason-class altimeter for nadir coverage
  - AMR-class radiometer (with possible high frequency band augmentation) to correct for wet-tropospheric delay

Manning's Equation to Estimate Discharge from SWOT

River discharge is often estimated on the ground by applying Manning's equation to fully rough, turbulent and uniform flows as follows:

$$Q=\frac{1}{n}AR^{2/3}s^{1/2}$$

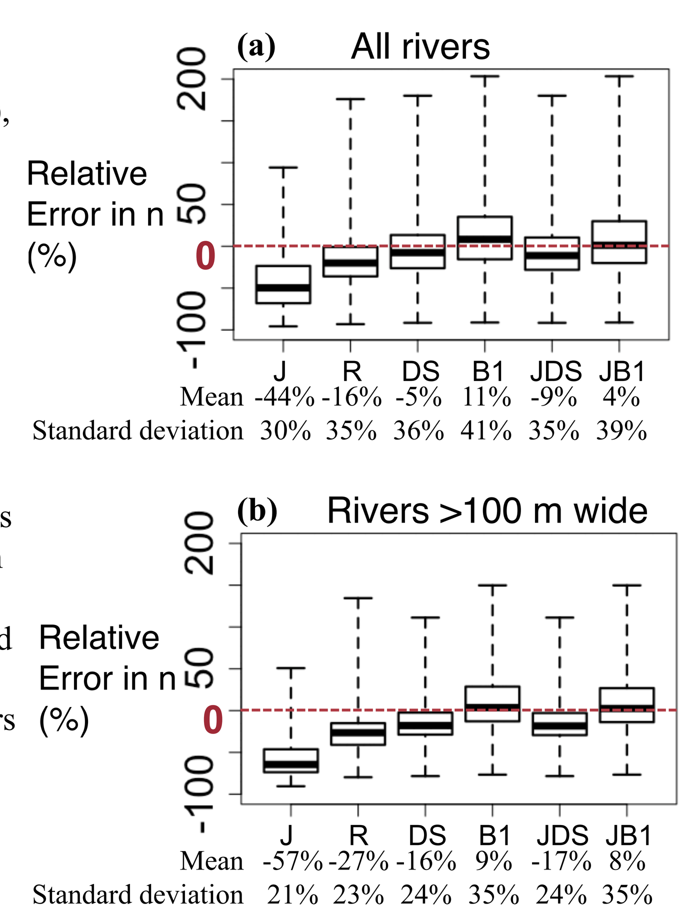
where n is the Manning's roughness, A is cross-sectional area (m²), R is hydraulic radius (m), and s is water surface slope.

LeFavour and Alsdorf (2005) applied this equation assuming a rectangular channel cross-section and river width much greater than depth to estimate flows for the Amazon River (to within <8% of in-situ gage estimates) from SRTM-derived slope and water heights, as well as ground-based estimates of channel bathymetry and river width. We can similarly apply this equation with water depth equal to a baseline water depth plus the change in water depth as:

$$Q=\frac{1}{n}w\left(z_0+dz\right)^{5/3}s^{1/2}$$

where w is width (m), z_0 is initial depth (m), dz is the temporal change in WSE measured by SWOT (WSE(t)-WSE(0)).

Figure 1. Box plot of relative errors in n for calculated using Jarrett (J), Riggs (R), Dingman and Sharma (DS), Bjerklie Model 1 (B1), a combination of J and DS (JDS), and a combination of J and B1 (JB1) regression models as applied to the ground-based data set. In combination models, J was used where s was between 0.002 to 0.04 and z was between 0.15 to 2.1 m, and DS or B1 was used elsewhere. (a) shows errors for all reaches in the data set, and (b) shows errors for only those reaches wider than 100m.



Water Center Annual Review
Feb. 17, 2010

Ground-based Data Set

The analyses presented here draw on a collection of reach-averaged channel properties of rivers in New Zealand, the Amazon, and the United States as compiled by Bjerklie et al. (2003) and provided to the authors by David Bjerklie. Summary statistics for the 1038 observations on 103 river reaches are tabulated here. Widths are greater than 100 m for 401 of these observations.

Reach-averaged Value	Mean	Standard Deviation	Minimum	Maximum
Discharge (m³/s)	1083	9056	0.01	283170
Width (m)	131	193	2.9	3870
Depth (m)	2.39	2.36	0.10	33.00
Slope (measured water surface or from topographic mapping)	0.0026	0.0052	0.000013	0.0418
Manning's n (calculated from measured discharge)	0.034	0.046	0.008	0.664

First Order Uncertainty Analysis

Assume that Manning's equation can be linearized using a first-order Taylor's series expansion, then the variance in Q about a mean "true" value due to measurement error can be written as:

$$E[Q(n,w,z,s)]\approx Q(E[n],E[w],E[z],E[s])$$

$$\therefore Var[Q]\approx ACA^T$$

where $A=\begin{bmatrix}Q'_n & Q'_w & Q'_z & Q'_s\end{bmatrix}$ and C is the covariance matrix.

If the terms are assumed to be independent, this becomes:

$$\frac{\sigma_Q}{Q}=\sqrt{\left(\frac{\sigma_n^2}{n^2}+\frac{\sigma_w^2}{w^2}+\frac{25(\sigma_{z_0}^2+2\sigma_{WSE}^2)}{9z^2}+\frac{\sigma_s^2}{4s^2}\right)}$$

Resulting relative discharge errors from applying this equation to the ground-based data set, assuming that $z_0 = 0.5*z$, are shown in Figure 2.

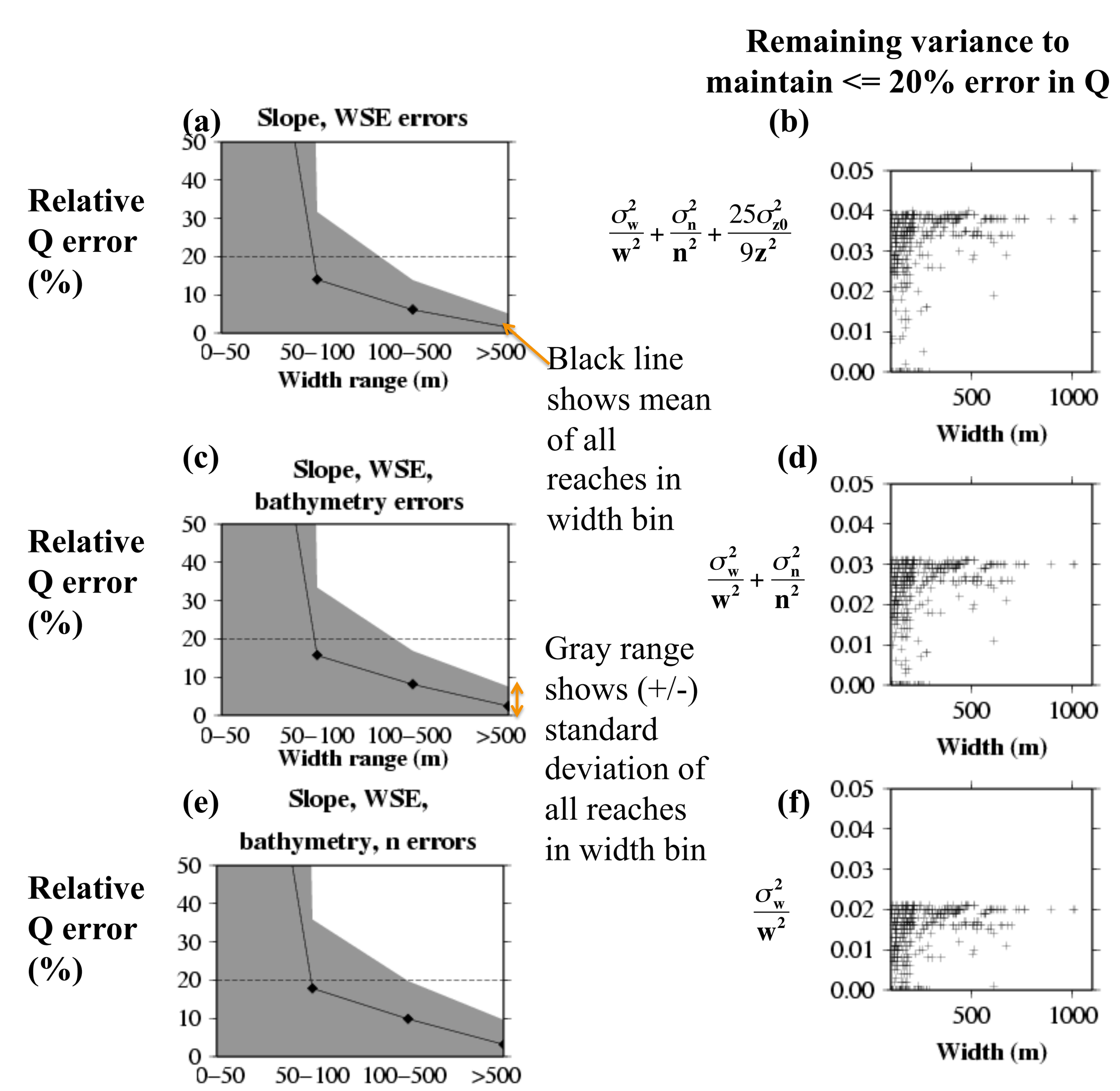


Figure 2. (a) Relative discharge error binned by width assuming SWOT accuracy requirements for slope (σ_s=1e-5) and WSE (σ_{dz}=0.10 m) and zero error in bathymetry, n, and width. (b) Remaining variance that can be distributed between bathymetry, n, and width without exceeding 20% error in Q. (c) Same as (a) except that error in bathymetry is included ($\sigma_{z0}=0.11*z_0$). (d) Remaining variance that can be distributed between n and width without exceeding 20% error in Q. (e) Same as (c) except that error in n (σ_n=0.10*n) has been included. (f) Remaining variance that can come from width without exceeding 20% error in Q.

Monte Carlo Error Propagation

To produce a more realistic distribution of errors, accounting for the fact that the actual error in a given term may fall above or below the 1σ error, we also used a Monte Carlo approach to modeling errors. For each observation in the ground-based data set, we generated 1000 perturbed realizations of discharge, representative of what might be estimated from SWOT observations, in which each variable was perturbed by a randomly generated error from the normal distribution with mean=0 and standard deviation=1σ. The relative errors in Q for all 1000 perturbed realizations of each observation were calculated and binned by width as before and are shown in Fig. 3. Sensitivity to initial depth is considered in Fig. 4.

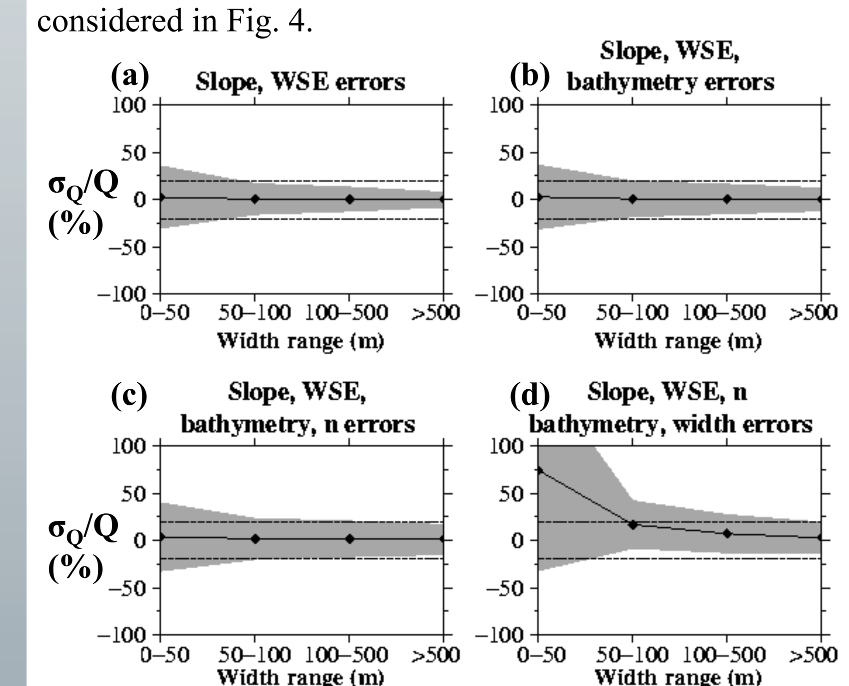


Figure 3. (a) Relative discharge error binned by width assuming SWOT accuracy requirements for slope (σ_s=1e-5) and WSE (σ_{WSE}=0.10 m) and zero error in bathymetry, n, and width. (b) Same as (a) except that error in bathymetry is included ($\sigma_{z0}=0.11*z_0$). (c) Same as (b) except that error in n (σ_n=0.10*n) has been included. (d) Same as (c) except that a constant 10 m bias in width has been included.

Figure 4. If errors in estimating the initial depth (effective bathymetry) are indeed relative rather than absolute, then the value of the initial depth impacts overall discharge error. In previous figures, the initial depth was assumed to be 50% of the one-time depth measurement in the ground-based data set. These panels show the relative discharge error due to slope, WSE and bathymetry errors (comparable to Figs. 2c and 3b) with the initial depth set to 20, 40, 60, and 80% of the one-time depth measurement. Lower initial depths (closer to baseflow) yield smaller Q errors.

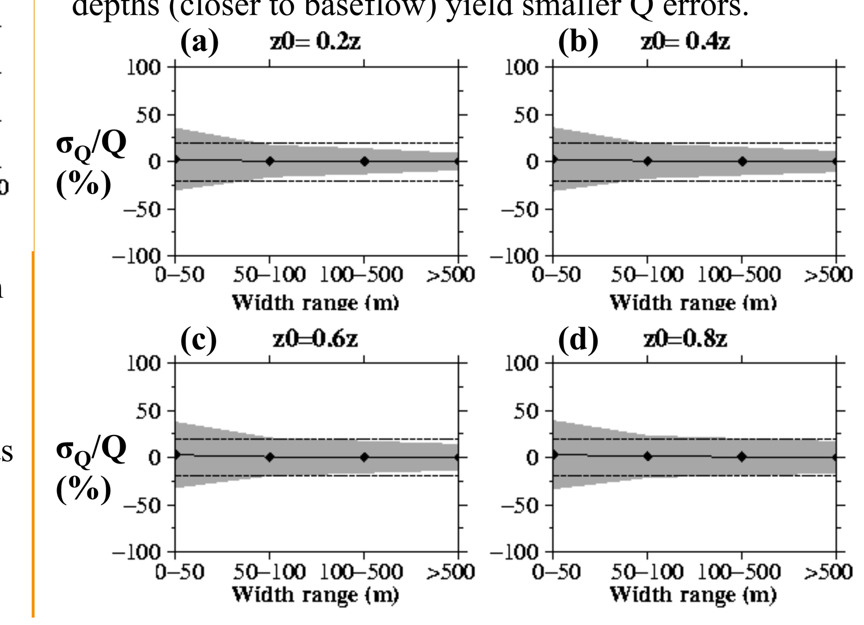
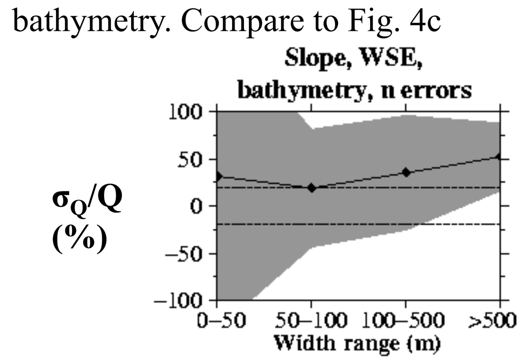


Figure 5. Relative discharge error, binned by width, where perturbed roughness is estimated by applying Dingman-Sharma regression model directly to perturbed slope, WSE, and bathymetry. Compare to Fig. 4c



Conclusions

- Discharge can be estimated by applying Manning's equation to SWOT-derived data and will be most accurate for large rivers, with accuracies at or near 20% for rivers wider than 100 m, assuming improved estimation of n.
- Discharge errors are highly sensitive to errors in water depth. Additional depth retrieval algorithms are under development and may lower the errors from this contribution. Estimating depth around an initial depth during low flows (enabled by collecting a long time series of data) can further limit the errors in SWOT-derived discharge.

Future work

- Width and roughness estimation algorithms require additional consideration, and investigations into deriving these from data assimilation into hydraulic models are on-going (Durand et al., 2008). The spatially distributed nature of SWOT measurements should be further exploited to improve roughness estimates.
- Error covariances between SWOT-derived variables and their impact on discharge error should be explored.

Acknowledgments

The authors would like to thank David Bjerklie for providing his compilation of ground-based reach-averaged data sets.

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